Module 8: Pupil Learning Activities for Addition beyond 20

CONTENT OF MODULE:
This module provides learning techniques to develop pupils’ number concept and counting skills up to 100 for Standard 1 and up to 1000 for Standard 2. Multiple strategies of developing number sense are introduced and practiced during the module including the use of counting forward and backward, skip counting and storytelling while using teaching aids such as the Number Line, Number Charts and Counters. These strategies develop the pupils’ comprehension of number concept, ability to see patterns, build fluency and improve memory capacity.

CORE CONCEPTS:
Pupils should be able to decompose or break a number apart and associate numbers using number bond facts to add numbers efficiently and with ease.

MATHS VOCABULARY:
Problem Story – a mathematical problem presented in the form of a story that captures the child’s imagination because it is humorous or relates to real life or creative; the problem may require only one step to solve or multiple steps based on the difficulty level.
Standard algorithm – for addition, it is when the problem is worked in a vertical format, with the digits aligned in columns by place value.

MODULE LENGTH: 4 hours for Core Concept and Activities + 2 hours of Lesson Planning

MODULE OBJECTIVES:
By the end of this module, teachers will be able to:
1. Develop pupils’ ability to add two-digit and three-digit numbers of varying difficulty using different approaches
2. Develop pupils’ ability to solve problem stories for addition beyond 20.
3. Develop pupils’ ability to add two-digit and three-digit numbers of varying difficulty using the standard algorithm

MATERIALS FOR THE SESSION:
1. INSET Module and a pen
2. Notebook, flip chart and markers (or use the blackboard for group work and a slate for individual work)
3. INSET Module 1, Standard 1 Syllabus and Standard 2 Syllabus
4. Teaching Aid Toolkit

LEARNING ENVIRONMENT FOR THE SESSION:
1. Review the ground rules established by the participants during the first meeting
2. Make revisions to the ground rules if required
3. Arrange the desks so that all participants can see and speak to each other
4. Feel free to ask questions
5. Always be supportive of your colleagues
6. Try to be creative and think about how ideas apply to your classroom
7. Put phones or pagers on silent mode

INSET COORDINATOR INSTRUCTIONS

- BEFORE MEETING WITH TEACHERS FOR THIS MODULE, LOOK AT THE ‘MATERIALS FOR THE SESSION’ AND MAKE SURE THAT YOU AND THE TEACHERS BRING THEM.
- BEFORE MEETING WITH TEACHERS FOR THIS MODULE, ENSURE THAT YOU AND THE TEACHERS HAVE READ THE MODULE IN ADVANCE.
- PLEASE HAVE THE TEACHERS SIGN THEIR SIGNATURE BELOW.

SAY TO TEACHERS:
‘Now we will read the introduction to this module. After the first teacher reads a paragraph aloud, they can call on another teacher to read the next paragraph.’

Date: [ ]
School: [ ]
District: [ ]
Region: [ ]
Start time: [ ]
INSET Coordinator name + signature: [ ]
Finish time: [ ]
Head Teacher name + signature: [ ]

Teacher name: [ ]
Signature: [ ]
Standard: [ ]

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Standard</th>
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<tbody>
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**REFLECTION**

**SAY TO TEACHERS:**

“Welcome to Module 5 of the INSET training for Counting. In this module we will learn how to incorporate teaching aids and pupil activities to develop the number concept up to 100 in Std.1 and 1000 in Std. 2. Before we begin, let us each share a success and a challenge faced when putting into practice the concepts and techniques discussed in the previous session. For each challenge that a participant mentions, let’s see if we can come up with a solution. Make sure to write down solutions that you find helpful or address the challenges that you identified.”

<table>
<thead>
<tr>
<th>READ ALOUD (5 MINUTES)</th>
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<tbody>
<tr>
<td>Since the last session we practiced one or more teaching technique to apply the following concepts developed during the module:</td>
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<tr>
<td>1. Incorporate the learning progression of introducing pupils to addition and subtraction into lesson planning.</td>
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<tr>
<td>2. Develop pupils’ conceptual understanding of addition and subtraction as increase and decrease of things using the number line.</td>
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<tr>
<td>3. Develop pupils’ fluency in mentally adding and subtracting up to 10 using number bonds up to 20.</td>
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<td>4. Understand the 3 ways to examine subtraction with pupils: adding on, take away, difference.</td>
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<tr>
<td>Take a moment to individually jot down a success as well as a challenge you experienced while conducting these lessons in you class.</td>
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<tr>
<th>WRITE INDIVIDUALLY (10 MINUTES)</th>
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<tr>
<td>- Write down individually a success and a challenge you experienced while applying these strategies in the classroom.</td>
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</table>
GROUP DISCUSSION (15 MINUTES)

- Share one of these experiences with the group.
- For each challenge, see if you can come up with solutions for your colleagues' challenges.
- During the discussion, write down solutions that pertain to the challenges you identified.

<table>
<thead>
<tr>
<th>Successes</th>
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<tr>
<td>(Describe the practice you have used and explain how you knew it was successful)</td>
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</table>

<table>
<thead>
<tr>
<th>Challenges</th>
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<tbody>
<tr>
<td>(Describe the practice and explain why it is challenging)</td>
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<table>
<thead>
<tr>
<th>Potential Solutions</th>
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</table>
“Let us reference the syllabus for Standards 1 and 2 and the example Scheme of Work in Module 1 to support our learning in this session.”

THINK – PAIR – SHARE (15 MINUTES)

- Review the activities for Number Operations in the 2015 Standard 1 Syllabus (pages 31-xx) and Standard 2 Syllabus (pages 27-xx).
- Review the example pupil activities for Number Operations described in the example Scheme of Work in Module 1 for Standard 1 and Standard 2.
- Reflect on the math concept and possible pupil activities for developing addition and subtraction skills.
- Turn to the person to your right and rapidly share example pupil activities that can be used in your classroom for this.

READ ALOUD (5 MINUTES)

In this module we will:

1. Develop pupils’ conceptual understanding of addition and subtraction as an increase and decrease of things using the number line
2. Improve pupils’ mastery in mentally adding and subtracting up to 10 using number bonds for Std. 1 and up to 20 for Std. 2
3. Learn ways to incorporate knowledge check activities into your lesson to assess pupils’ learning progress

Learning Progression for Addition and Subtraction

In the previous module, we discussed the learning progression for addition and subtraction as illustrated by the diagram below:
We also practiced approaches for adding and subtracting with single-digit numbers within 20. Below, you will see an example of one or more of the following key teaching strategies for each type of addition exercise:

1) Band of Ten
2) Place Value Chart with Bundles of Sticks
3) Place Value Chart with Number Discs
4) Number Bonds
5) Number Line
6) Standard Algorithm

Many of the visual representations of the addition problem (Number Line, Number Bonds) rely on pupils' ability to decompose and associate numbers.

In order to make the addition problems have real-life meaning and value, it is encouraged to introduce the exercises within the context of a problem story.

**What is a Problem Story?**
A problem story is a mathematical problem presented in the form of a story that captures the child's imagination because it is humorous or relates to real life or engages pupils' imagination. A problem story is different from storytelling, which was introduced in Module 6. Storytelling captures pupils' attention while
introducing an idea or concept in maths. A problem story requires them to solve a question posed within the story. The problem may require only one step to solve or multiple steps based on the difficulty level. The advantage of presenting problems in the context of a story is because it is easier to discuss in terms of real things and makes learning addition or any mathematical operation meaningful for children.

Adding a single-digit number to a two-digit number without ‘carry’

Once pupils have mastered the number bonds for adding single-digit numbers, the next level of difficulty is to add a single-digit number to a two-digit number without ‘carry.’ First introduce this type of addition problem in the context of a story. For example, you can narrate the following problem story to the class:

‘Mohammed read 22 pages of his Swahili book yesterday. Today, he read another 5 pages. How many pages did he read in total?’

This is a story that pupils can easily imagine and find the answer without any formal mathematics. They can solve by counting on from 22 to 27 using their 5 fingers. However, you should also ensure pupils use alternate approaches such as the examples below:

- **Use the Band of Ten**
  The Band of Ten teaching aid can be used to develop quick strategies to add. For instance, children may observe that there are 7 counters in the third Band of Ten because there are 3 counters missing. Also, without counting one by one, children should be able to know that 20 in the first two Bands of Ten.

```
\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|}
\hline
\text{Red} & \text{Red} & \text{Red} & \text{Red} & \text{Red} & \text{Green} & \text{Green} & \text{Green} & \text{Green} & \text{Green} \\
\hline
\end{array}
\]
```

- **Use the Number Line**
  Pupils should learn to mentally apply decomposition and association to produce the following without using counters.

\[
22 + 5 = (20 + 2) + 5 = 20 + (2 + 5) = 20 + 7 = 27
\]

When children are using the number line, we can identify which child is still counting on by ones from those that are skip counting by five.

- **Use Number Bonds**
This approach strengthens pupils’ ability to decompose and associate using number bonds that they are most comfortable with. After representing addition problems this way, the teacher can follow up with oral exercises to build pupils’ mental calculation abilities.

![Number Bonds Diagram]

By using different approaches you ensure that pupils of differing learning styles (listening, seeing and manipulating objects) develop conceptual understanding of addition. Also, you prepare them for more difficult addition exercises.

**Adding a single-digit number to a two-digit number with ‘carry’**

Now pupils should be introduced to a more difficult type of problem that is adding a single-digit number to a two-digit number with ‘carry.’ The problem story sets the context:

“The chalk box had 28 pieces of chalk in it. Mr. Katabalo had a few more in his pocket. When he pulled them out to check, there were 5 more pieces. How many pieces of chalk were there altogether?”

- **Use the Number Line**
  On the number line, pupils can approach this by using their knowledge of number bonds. For example,

  \[
  28 + 5 = 28 + 2 + 3 = 30 + 3 = 33
  \]

  This corresponds to jumping to the first number, then jumping to the nearest ten above it, then jumping the rest of the way. The mental strategy essentially involves calculating the size of this last jump.

- **Use Number Bonds**
  Different strategies should be recognized as equally valid and their benefits discussed with pupils. In particular, pupils should be introduced to the process used in the standard algorithm in an informal
way. For example,

\[
28 + 5 = 20 + 8 + 5 = 20 + 13 = 33
\]

Note that this approach reduces to a decomposition and two applications of adding two single-digit numbers, with one of the additions taking place in the tens column.

- **Place Value Chart with Bundles of Sticks or Number Discs**
  Another strategy is to use the place value chart and bundles of sticks or number discs. First begin by representing 28 using 2 tens and 8 ones.

<table>
<thead>
<tr>
<th>PLACE VALUE CHART</th>
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<tbody>
<tr>
<td>TENS</td>
</tr>
<tr>
<td><img src="image1.png" alt="Tens" /></td>
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<tr>
<td>2</td>
</tr>
</tbody>
</table>

Then add 5 ones to the ones column. As you do that, remind pupils that we cannot have more than 9 ones in the ones column.
So once you have 10 ones, it must be replaced by a bundle of ten and placed in the tens column resulting in the answer: 33.

Adding two two-digit numbers with no carrying involved
Mental strategies for adding two-digit numbers usually involve decomposing one of them and reducing the problem to one, or a combination, of the cases already discussed above. Consider the example 24 + 15.

- **Use the Number Line to first add ones then add tens**

  This approach, which is formalized in the standard algorithm, corresponds to
  
  \[
  24 + 15 = (24 + 5) + 10 = 29 + 10 = 39
  \]

  On the number line, this corresponds to skip counting as illustrated below:
• **Use the Number Line to first add tens then add ones**

This involves the calculation $24 + 15 = 24 + 10 + 5 = 34 + 5 = 39$

This is a valid approach. Developmentally it usually comes before the previous technique. It is complex when applied algorithmically, so it is important that pupils who naturally discover this method also understand the previous approach. On the number line this corresponds to implementing the second and third jumps above in the opposite order.

![Number Line](image1)

• **Use Number Bonds**

This involves the calculation $24 + 15 = (20 + 10) + (4 + 5) = 30 + 9 = 39$ where you decompose 24 and 15 and associate the tens and the ones.

![Number Bonds](image2)

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**Adding two two-digit numbers with 'carry'**

The next level of complication involves introducing 'carry'. We illustrate various techniques using $28 + 15$.

• **Use Number Line or Number Bonds**

The numbers can be decomposed and associated in different ways; each pupil can think of the addition problem in these different ways, all of which are correct. Some are less efficient than others but nonetheless correct. This type of exploration sets the stage for introducing the standard algorithm, which is the most efficient way of adding large numbers with the exception of using a calculator or a computer.

The teacher should encourage this type of exploratory thinking and the number line or number bonds can
facilitate this.

Pupils can first add ones then add tens:

\[ 28 + 15 = 28 + 5 + 10 = 33 + 10 = 43 \]

Alternatively they can first add tens then add ones:

\[ 28 + 15 = 28 + 10 + 5 = 38 + 5 = 43 \]

Or

\[ 28 + 15 = 28 + 2 + 13 = 30 + 13 = 43 \]

Or

\[ 28 + 15 = 23 + 5 + 15 = 23 + 20 = 43 \]

- Use Place Value Chart with Bundles of Sticks or Number Discs
Adding with three-digit numbers

In Standard 2, addition should include three-digit numbers. The step-by-step progression continues to apply. First introduce problems of adding a single-digit or two-digit number to a three-digit number without carry, then with carry. Increase the difficulty with adding a two three-digit numbers without carry and finally with carry. Use problem stories to set the context of the addition exercise and then explore with Number Bonds or Place Value Charts. Constantly ensure correct use of vocabulary (add, addition, equals) throughout the learning process. However, the symbolic way of representing the problem \( (22 + 5 = 7) \) and the use of the standard algorithm should be introduced only after there is some level of conceptual understanding.

GROUP DISCUSSION (30 MINUTES)

- What were some of the new or important ideas that you marked with an exclamation (!) point?
- What were some of the unclear ideas that you marked with an exclamation (?)?
- What were new concepts that you circled?

ACTIVITY – ADDITION BEYOND 20

SAY TO TEACHERS:
“Now you will do an activity with your group. First take 5 minutes to silently read the example lesson that illustrates how to use number bonds to develop addition skills.”

ACTIVITY – ADDITION BEYOND 20

SILENT REFLECTION (5 MINUTES)

- Read the example lesson plan below silently and think about how you can try it in your classroom
- Write down any doubts or questions in your notebook to share with the group at the end of this session

3 LESSON PLANS –
ADDING TWO 2-DIGIT WITHOUT CARRY problem story, number bonds, answer to slate
ADDING TWO 2-DIGIT WITH CARRY problem story, place value and number bonds
**Learning Objective:**
*Add with total not exceeding 1000*

**Lesson Objective:**
*Add a two-digit and three-digit number with carry*

**Material:**
*Activities: In pairs*

**Vocabulary or Phrase:**
*add, total, +, =*

**Steps to follow:**
1. **Introduction** (5 minutes):
   - Problem Story
2. **New Knowledge** (10 minutes):
   - Using the cards from the introduction, display picture cards of the same object but separated into two groups on the basis of a clearly visible feature. For example, the object is an orange, feature is that one
   - Remove the picture cards and draw the two different number bonds of the same number
   - Repeat at least three times with different groups of picture cards for different number bonds
3. **Lesson Knowledge Check** (10 minutes):
   - Check pupils’ understanding by playing ‘Clap your hands Tap your feet’
   - Explain the game that if the teacher claps her hands, the pupils should write two commutative number bond

**ROLE PLAY (20 MINUTES)**

In a group, practice the lesson. One teacher can play the role of the teacher and all the others can pretend to be pupils in Standard 1.

**TURN AND TALK (10 MINUTES)**

- After trying the activity, turn and talk to the person to your right about the experience. Some questions to reflect on:
  - Do you think this activity is suitable for your classroom?
  - Will you practice it in your class?
  - What challenges do you think you will encounter in trying it in the class?

**CORE CONCEPT – STANDARD ALGORITHM FOR ADDITION**

**READ ALOUD (15 MINUTES)**

3. One teacher should start reading out loud. After he/she finishes the first paragraph, he/she can call out another teacher’s name so that they read the next paragraph.
4. While reading:
   - Put an exclamation point (!) next to anything that you find important
What is the Standard Algorithm?

Only after pupils have understanding of the concept of carrying using place value, should the standard algorithm be introduced. The standard algorithm is the last strategy to be taught as it is the highest form of abstraction and the most difficult to understand. The word ‘algorithm’ originally meant the art of calculating by means of nine fingers and a zero. But we use it to mean a procedure for solving a mathematical problem in a finite number of steps by repeating an operation. Pupils do not need to know the term algorithm, but they do need to understand the specific steps to follow. In the standard algorithm for addition, the problem is worked in a vertical format, with the digits aligned in columns: one column for each place value.

Consider 345 + 47:

```
  3 4 5
+  4 7
```

Steps of the Standard Algorithm

First, the ones are added. If the sum is 10 ones or more, it is renamed as tens and ones and ones are written down under the line in the ones place (under the column of ones) and the ten is represented above as 1 ten in the tens column.

```
  3 4 5
+  4 7
  1 5 2
```

Then the tens are added, including the ten that was carried over from the previous step. If their sum is 10 tens (100) or more, it is carried as hundreds and tens and the tens written down in the tens place under the line.

```
  3 4 5
+  4 7
  5 6 2
```

The hundreds are then added, including any carried over tens.

```
  3 4 5
+  4 7
  8 0 2
```
ACTIVITY – STANDARD ALGORITHM FOR ADDITION

3 LESSON PLANS –
ADDING TWO 2-DIGIT WITHOUT CARRY problem story, number bonds, answer to slate
ADDING TWO 2-DIGIT WITH CARRY problem story, place value and number bonds
ADDING 3-DIGIT TO 2 DIGIT WITH CARRY problem story, number bonds

LESSON PLANNING

SAY TO TEACHERS:

“To improve pupils’ learning, it is very important that teachers are able to practice the teaching techniques they learn from INSET in their classrooms. For this, it would be beneficial to develop the lesson plans together as a group instead of individually. When we support each other through this process of lesson planning, we will be able to design better quality lessons. So let us dedicate at least 2 hours to lesson plan together. Let us decide now when we will meet next to complete this component of the INSET for this module.”

JOINTLY DETERMINE WHEN THE TEACHERS WILL MEET AGAIN TO COMPLETE THE LESSON PLANNING SECTION OF THIS MODULE. WHEN YOU MEET AGAIN TO WORK ON THIS SECTION, WALK AROUND AND SEE IF TEACHERS NEED HELP WITH THE PLANNING.

LESSON PLANNING (2 HOURS)

To improve pupils’ learning, it is very important that teachers are able to practice the teaching techniques they learn from INSET. Therefore, it is important to set aside at least 2 hours to lesson plan together with other teachers.

• Set a time to meet in order to complete this activity before continuing on to the conclusion of today’s session.
• When you meet for the lesson planning, first review the core concepts covered in this module.
• Then as a group, plan two different lessons that incorporate at least one of the teaching techniques or core concepts covered in this module.
• As a group, determine the appropriate learning objectives for your lessons based on the concepts covered in this module.
• Complete the two lesson plans using your lesson plan template.
• You are expected to implement the lesson plan in your classroom with your pupils
• Before beginning the new module, you will have an opportunity to share your challenges and successes in trying out the lesson plan.

CONCLUSION

WRITE INDIVIDUALLY (15 MINUTES)

Please fill in the following form to record your appraisal of today’s module. After you are finished, rip this page out and give it to your INSET Coordinator. Please be honest with your answers because your feedback will help to improve school based INSET in the future.

<table>
<thead>
<tr>
<th>Marking Scheme for the INSET Appraisal:</th>
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<tbody>
<tr>
<td>0 points: I completely disagree with the statement</td>
</tr>
<tr>
<td>1 point: I partially disagree with the statement</td>
</tr>
<tr>
<td>2 points: I partially agree with the statement</td>
</tr>
<tr>
<td>3 points: I completely agree with the statement</td>
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</tbody>
</table>

INSET Appraisal Form:
**OVERALL SUCCESSES AND CHALLENGES OF TODAY’S SESSION AND COMPLETE THE FORM BELOW.**

<table>
<thead>
<tr>
<th>Overall successes from this session:</th>
<th>Overall challenges from this session:</th>
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</thead>
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</tbody>
</table>

School: ______________________    District: _______________________
Region: _______________________
Appraisal for Module # ________     Topic of Module:

Number of teachers who participated: ________         Did the Head Teacher participate:   Yes/No
Was the INSET Coordinator present to facilitate:   Yes/No

Read the statements below and tick the box that indicates whether your answer:

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<th>0</th>
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1. The Core Concepts of today’s module was very clear. I feel like I have a very good understanding of the topics.
2. This module had many useful and relevant strategies that I will try in my class.
3. The amount of time it took to complete this module was appropriate. It did not feel too long.
4. This module prompted a lot of interesting discussion and reflection.
5. The INSET Coordinator was prepared for the session – he/she has clearly read the module and had all the materials ready.
6. The INSET coordinator effectively facilitated discussion – he/she knows how to get people talking and how to help with answers.
7. The INSET coordinator knows how to maintain a good group dynamic – he/she makes sure that teachers are supportive, collegial and energised.
8. The INSET coordinator knows how keep teachers motivated – he/she follows up with teachers who are absent/late and reminds us of why INSET is important.

- Close the session by setting the meeting time and date for the Lesson Planning session for this module and the meeting time and date for the new module.